In connection with the use of supersonic nozzles to create lasers, the question arises of the optimum parameters of the nozzle and the gas mixture from the aspect of obtaining the greatest population inversion of the energy levels of internal degrees of freedom of molecules of the working gas and the greatest output power of the lasers. A rather complete concept of the kinetic processes taking place during the escape of a relaxing gas mixture containing carbon dioxide through a supersonic nozzle has now been developed on the basis of calculated and experimental data. In [1-4] the problems of optimization of the parameters of a $\mathrm{CO}_{2}-\mathrm{N}_{2}-\mathrm{H}_{2} \mathrm{O}-$ He mixture and of the shape of the nozzle were set up and solved in a one-dimensional steadystate formulation. The influence of the two-dimensionality of the stream in an optimum nozzle on the laser characteristics is studied in the present report. The method of through calculation suggested in [5] is used to calculate the two-dimensional flow of a relaxing gas.

We consider the motion of a $\mathrm{CO}_{2}-\mathrm{N}_{2}-\mathrm{He}$ gas mixture with allowance for kinetic processes described by three nonequilibrium temperatures (see [1, 3, 6]). An analysis of the dimensionality of the parameters of the problem shows (see [5]) that the flow is determined by the following parameters:

$$
T_{0}, p_{0} L_{0}, \beta_{j}, x_{j} / L, h_{0} / L_{0}, L / L_{0}, \xi_{1}, \xi_{2}
$$

here $\mathrm{T}_{0}$ and $\mathrm{p}_{0}$ are the stagnation temperature and pressure of the gas; $\xi_{1}$ and $\xi_{2}$ are the molecular contents of carbon dioxide and nitrogen ( $\xi_{3}=1-\xi_{1}-\xi_{2}$ is the helium content); $L_{0}$ is the characteristic length; $h_{0}$ is the half-height of the critical cross section; $L$ is the length of the nozzle; $\beta_{j}$ are parameters of the angle type, assigned at the points $\mathrm{x}_{\mathrm{j}} / \mathrm{L}$ and determining the shape of the nozzle. In [2,3] optimization was carried out for the case when the nozzle contour was assigned through the equation $y s=h_{0}\left(1+\omega\left(x^{\prime}\right)\right)$, where $x^{\prime}$ is the distance along the x axis normalized to the length L of the nozzle, i.e., $\mathrm{x}^{\prime}=\mathrm{x} / \mathrm{L} ; \omega\left(\mathrm{x}^{\prime}\right)$ is a dimensionless function which is determined by the values of its derivatives $\beta_{0}, \beta_{1}$, and $\beta_{2}$ with respect to $x^{r}$ at the points $\mathrm{x}^{\prime}=\mathrm{x}_{\mathrm{j}} / L$, equal to $0,1 / 9$, and $4 / 9$, respectively, and by the conditions $\omega(0)=0$ and $d \omega / \mathrm{dx}^{\boldsymbol{r}} \mid \mathrm{x}^{\prime}=1=0$. Between the points $x_{j}^{\prime}$ the function was interpolated by parabolas. The solution of the one-dimensional problem does not depend on $\delta=h_{0} / L_{0}$, and therefore if one takes the length $L$ of the nozzle as $L_{0}$ and considers fixed values of the initial pressure $p_{0}$, then the list of parameters with respect to which the optimization takes place appears as follows in the indicated case: $\mathrm{T}_{0}, \mathrm{p}_{0} \mathrm{~L}, \xi_{1}, \xi_{2}, \beta_{\mathrm{j}}$.

The optimization of the specific power, i.e., the power normalized to the gas flow rate, was carried out at a fixed pressure $p_{0}$; and therefore we determined $T_{0}, L, \xi_{1}, \xi_{2}$, and $\beta_{j}$. The two-dimensional flow of the resulting mixture in a nozzle determined in this way depends in addition on the parameter $\delta$. For small $\delta$ the flow can evidently be described satisfactorily through the one-dimensional theory. An increase in $\delta$ leads to an increase in flow rate and hence in the total output power of the laser. But with an increase in $\delta$ the twodimensional character of the stream is manifested to a greater extent. From the equation for the nozzle contour it is seen that the value of the ratio $h_{0} / L$ determines the angle $\theta$ of inclination of the tangents to points of the contour, other fixed optimum parameters $\mathrm{T}_{0}, \mathrm{~L}, \xi_{1}, \xi_{2}$, and $\beta_{\mathrm{j}}$ being equal:

$$
\operatorname{tg} \theta_{j}=d y_{s} / d x=\delta d \omega / d x^{\prime}=\delta \beta_{j} .
$$

Therefore, to deliver practical recommendations on nozzles and mixtures one must make two-dimensional calculations of flows with variation of the height of the critical cross section with fixed optimum parameters determined from the one-dimensional theory. To illustrate this conclusion, let us discuss the results of optimization of the specific power at $p_{0}=10 \mathrm{~atm}$. In this case $\xi_{1}=10.8 \%, \xi_{2}=59.16 \%, \xi_{3}=30.04 \%, \mathrm{~T}_{0}=2810^{\circ} \mathrm{K}$, $\mathrm{L}=5.23 \mathrm{~cm}, \beta_{0}=170.71, \beta_{1}=37.41$, and $\beta_{2}=18.94$ [3]. The presence of a large curvature of the contour can

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lead to sharp degradation of the laser parameters in the two-dimensional analysis in connection with the formation of compression shocks in the stream. Therefore, instead of the indicated $\beta_{j}$ we calculated variants with $\beta_{0}=64.8, \beta_{1}=37.41$, and $\beta_{2}=18.94$, which leads to a $10 \%$ loss in power in comparison with the optimum value according to the one-dimensional theory.

The results corresponding to a value of $\delta=0.0066$ (in which case the aperture half-angle is $\theta_{0}=24^{\circ}$ ) are presented in Fig. 1. The distance from the axis of symmetry of the nozzle is laid out along the abscissa, with $y=1$ corresponding to a point on the contour of the nozzle and $y=0$ to the axis of symmetry; the distributions of temperatures $T$ and $T_{i}$ (curves 1-4) and of the gain (curve 5) along $y$ are shown; the average over $y$ of the gain and the value of $\mathrm{k}_{0}$ obtained through a solution of the flow problem in a one-dimensional statement are also indicated here by the segments 6 and 7. The difference between the average value $\langle k\rangle$ and the gain $k_{0}$ is small, and the relative difference is $\left|\left(\langle\mathrm{k}\rangle-\mathrm{k}_{0}\right) / \mathrm{k}_{0}\right|=13 \%$. The variation of the coefficient k along y is connected with the variation of the gasdynamic variables in a cross section.

Analogous results for $\delta=0.01144$, which corresponds to an initial aperture angle $\theta_{0}=36^{\circ}$, are presented in Fig. 2. In this case the degree of expansion $S$ of the stream as a function of the size of the critical cross section $\delta$ varies in the form $S=\delta 111.56$. A shock wave, behind which the translational temperature $T$ is high, develops in the stream. The presence of a shock wave does not cause marked changes in the vibrational temperatures $T_{3}$ and $T_{4}$, but the temperature $T_{2}$ varies markedly. The amount of gain falls to zero near the nozzle wall. In this case the relative departure of $\langle\mathrm{k}\rangle$ from $\mathrm{k}_{0}$ is $\left|\left(\langle\mathrm{k}\rangle-\mathrm{k}_{0}\right) / \mathrm{k}_{0}\right|=18 \%$. The value of T in the cross section $x^{\prime}=0.6$ is marked by a dash-dot line in Fig. 2.

Figure 3 gives a concept of the intensity of the shock as a function of the initial aperture half-angle $\theta_{0}$. Here the distance from the axis of symmetry of the nozzle is laid out along the abscissa (as in Figs. 1 and 2) while curves $1-5$ correspond to the distribution of the translational temperature $T$ at values of $\theta_{0}$ equal to $24.3,31.4,36.1,42.4$, and $50.8^{\circ}$, respectively. The values of $T$ for the maximum half-angle $\theta_{0} \approx 20^{\circ}$ at which a shock wave does not develop under these conditions are indicated by a dashed line. It is seen that as the initial angle $\theta_{0}$ increases the intensity of the shock increases and the relative y coordinate of the formation of the shock increases, i.e., the shock "presses" against the nozzle wall.

The distribution of the average gain $\langle k\rangle$ along the nozzle axis $x^{\prime}$ for the same values of the initial angle $\theta_{0}$ is shown by the curves in Fig. 4 with the previous numbering. The distribution of $k_{0}$ according to the onedimensional theory is indicated by a dashed line.


Fig. 5
In Fig. 5 curve 1 is plotted from the values of the average gain $\langle k\rangle$ in the cross section $x^{\prime}=1$ as a function of $\delta=h_{h 0} / L$. The dash-dot line refers to the value of $k_{0}$. The angles $\theta_{0}$ for different $\delta$ are determined from curve 2. The dashed curve gives the ordinate of the location of the compression shock in the cross section $X^{\prime}=1$.

At small values of $\delta$ one observes good agreement between curve 1 and the dash-dot line, and in some section of variation the average coefficient $\langle k\rangle$ exceeds the value $k_{0}$, i.e., in a two-dimensional stream better conditions occur for the formation of an inversion than are obtained in one-dimensional calculations. Then a decrease occurs in the coefficient $\langle k\rangle$, which is due to the appearance of shock waves in the stream.

With an increase in the flow rate the intensity of the shock grows, which is seen from Fig. 3, and the optical gain falls sharply behind it. But the decrease in the relative thickness of the deactivation zone compensates for this fall, and the average value $\langle\mathrm{k}\rangle$ at the nozzle exit grows.

Thus, the results of the calculations in the given case permit one to state that the practical realization of the optimum parameters of the nozzle and the gas mixture obtained from the one-dimensional theory will be very interesting.

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